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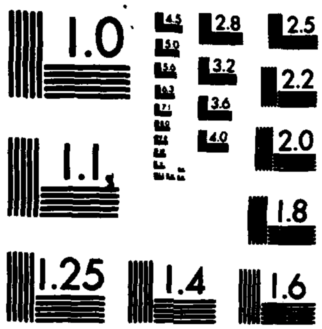

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THE USE OF BOWRING'S ALGORITHMS
FOR HYDROGRAPHY AND NAVIGATION

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ABSTRACT

Concise, efficient, noniterative direct and inverse geodetic position computation algorithms for short geodesics on the ellipsoid have been published by B.R. Bowring (1981). These algorithms are ideal for geodetic surveying applications, and their sub-millimeter accuracy has been verified for geodesics up to 150 km long (Vincenty 1982). The intent of this paper is to present results of an investigation of the behavior of Bowring's algorithms over longer geodesics and to ascertain their applicability to hydrography and general navigation.

The preoccupation of geodesists with direct and inverse position computation on the terrestrial ellipsoid has a long and distinguished history. Gauss, Bessel, Helmert, Puissant, Rainsford, McCaw, and Sodano are all prominent names associated with formulas and algorithms developed for the solution of what in the German technical literature used to be called "die geodaetische Hauptaufgabe" (the principal geodetic problem). With the advent of electronic computers, this work of giants was given a capstone by T. Vincenty with his optimal adaptation for automatic computation of the globally accurate Bessel-Helmert-Rainsford iterative algorithms (Vincenty 1975, 1976).

The "direct" problem can be posed as follows: Given the position (latitude and longitude) of a point on the reference ellipsoid (the "standpoint"), as well as the orientation (forward azimuth) and length of a geodesic line emanating from it, compute the position (latitude and longitude) of the terminal point of that geodesic line (the "forepoint") and its back azimuth. The "inverse" problem, as can be expected, is the converse of the direct problem: Given the coordinates of two points on the reference ellipsoid, compute the length of the geodesic line joining them, as well as the forward and back azimuths at the respective endpoints, which in this case are arbitrarily taken to be the standpoint and the forepoint.

Vincenty's direct and inverse position computation algorithms are efficient and accurate to a fraction of a millimeter for short and long geodesics alike, ranging in length from a few centimeters to just under half-way around the world. As such, they are yardsticks against which the performance of other direct and inverse position computation algorithms are to be measured. However, they are iterative, which is to say that the number of steps in a solution can vary depending on the geometry of the individual problem.

Since convergence is very fast (two or three iterations is the norm), the iterative nature of Vincenty's algorithms is hardly a consideration in non-realtime applications run on present-day powerful minicomputers and mainframes; in fact, it has been shown that Vincenty's algorithms execute faster than either Sodano's or Andoyer-Lambert's noniterative long-line counterparts. There are, however, applications for which one would intuitively prefer the noniterative solutions of the direct and inverse

position computation problems, solutions which would not have the complexity of the existing long-line noniterative algorithms, result in even more compact code, and execute faster than Vincenty's algorithms, and still deliver the desired accuracy.

One such application is the computation of geodetic survey work on the ellipsoid (as opposed to plane coordinates) implemented on a portable microcomputer (e.g., the surveyor's field computer). Here one is typically faced with very limited memory and the need for compact code, with speed of execution being an important but secondary consideration. Sub-millimeter computational accuracy is required in this application; however, the line length is limited by intervisibility and seldom exceeds 50 km.

Another such application occurs in hydrographic surveying; i.e., the realtime computation of the position of a survey vessel with respect to shore control stations, when one wishes to work with the reference ellipsoid rather than with a map projection. Here the maximum line length will vary from 50 km for line-of-sight positioning systems, to 300 km for medium-range systems such as Raydist or Argo, to 1500 km for a long-range system such as LORAN-C. On the other hand, the computational accuracy requirement can be proportionately relaxed by two, three, and four orders of magnitude (compared with the geodetic surveying case), depending on the scale of the survey and positioning method used; e.g., 0.1 m for short-range control, large-scale surveys (for harbor approach charts), 1 m for medium-range control, medium-scale surveys (for coastal sailing charts), and 10 m for long-range control, small-scale surveys (for general sailing charts). Since in a realtime application the respective algorithms must execute within an assigned time slot measured in milliseconds, speed of execution is the primary consideration in this instance.

Recently, B. R. Bowring of Surrey, England, developed and published very elegant noniterative algorithms for the direct and inverse position computation over "short" geodesic lines up to 150 km (Bowring 1981). These "quasi-spherical" formulas are remarkably concise and accurate within their intended range of application; they very likely represent the last word in streamlining the solution of the "principal geodetic problem." Bowring's algorithms lend

themselves admirably to the first application outlined above, i.e., computation of geodetic survey work. They were successfully used by this writer as the basis of a powerful and efficient geodetic package of programs implemented on the Hewlett-Packard HP-9815A desktop computer (Taylor 1981).

The purpose of this paper is to present the results of an investigation as to the extent to which Bowring's algorithms are sufficiently accurate to support the second application mentioned above, i.e., the realtime positioning of a surface vessel for hydrographic surveying or precise navigation purposes. This investigation evaluated the total position error produced by Bowring's algorithms over a large number of geodesic lines emanating from standpoints located at seven representative latitudes (0, 15, 30, 45, 60, 75, and 89 degrees), in nine representative azimuths (0, 30, 45, 60, 90, 120, 135, 150, and 180 degrees), and of lengths ranging from 50 to 4000 km (preliminary computations indicated this distance to be the usable limit). In all, 4284 cases were computed.

As a first step in every case, the coordinates of each forepoint were computed using the precise Vincenty's direct algorithm. This step was then repeated using Bowring's direct algorithm, and the distance separating the two sets of coordinates was taken as the total position error of Bowring's direct algorithm. Next, Bowring's inverse algorithm was used to recover the length and forward/back azimuth of the geodesic line between the given standpoint and computed precise forepoint coordinates. The resulting length and forward azimuth were then used as arguments in Vincenty's direct algorithm to compute another set of forepoint coordinates, and the distance separating the two sets of coordinates was taken as the total position error of Bowring's inverse algorithm.

The total position errors, obtained in meters, were shown as proportional errors relative to the length of the geodesic line, in parts per million (ppm). Inspection of the tabulations confirmed that Bowring's direct and inverse algorithms are well balanced with respect to accuracy, as the corresponding errors in any given case were always very nearly equal. For each of the 63 geodesic lines computed at distance increments from 50 to 4000 km (9 radial lines from each of 7 standpoint latitudes), the tabulations were searched to determine the distances at which total position errors

exceed the thresholds of 0.0001 m, 0.001 m, 0.01 m, 0.1m, 1 m, 10 m, and 100 m; and in terms of relative error, the thresholds of 0.1 ppm (1:10,000,000), 0.2 ppm (1:5,000,000), 1 ppm (1:1,000,000), 2 ppm (1:500,000), 10 ppm (1:100,000), and 20 ppm (1:50,000).

As could be expected from the nature of the problem, the worst performance for radial lines emanating from each standpoint was along the meridian, with progressively better performance along geodesics in azimuths away from the meridian. For each of the seven standpoint latitudes, this worst-case performance was taken as the upper bound of the total position error to be expected of Bowring's direct and inverse algorithms over any geodesic line having an endpoint at that latitude. The resulting information is portrayed graphically in Figures 1 and 2.

By inspection of the log-linear graph of Figure 1, it is clear that even in the worst possible case (geodesic line on or near the meridian originating at or near the latitude of 45 degrees), Bowring's algorithms meet the computational accuracy requirements of both geodetic survey work and of surface vessel position fixing consistent with the accuracy of shore-based positioning systems likely to be used for hydrographic surveying and precise navigation purposes. The total position error produced by either the direct or the inverse algorithm is guaranteed to be less than 0.001 m up to 100 km, less than 0.1 m up to 500 km, and less than 10 m up to 1500 km. One notes that the error curves are symmetrical about the latitude of 45 degrees, and that progressively better accuracy performance is obtained along geodesic lines originating in both lower and higher latitudes, as well as along geodesics in azimuths away from the meridian.

It is also interesting to note that on the log-log graph of Figure 2, the relative error in parts per million as a function of line length is linear. This quite unexpected result clearly suggests an empirical formula for the global upper bound of the total position error produced by Bowring's algorithms. By considering the worst-case performance, the following empirical relationship (Equation (1)) for the maximum relative error in parts per million (M_{ppm}) as a function of geodesic line length in kilometers (D_{km}) can be derived:

$$M_{ppm} = 7.17 \times 10^{-9} D_{km}^{2.86} \qquad (1)$$

An expression for the absolute maximum error can be derived by multiplying Equation (1) by distance. Taking into account the conversion of distance units to meters, the following equation results:

$$M_m = 7.17 \times 10^{-12} D_{Km}^{3.86} \qquad (2)$$

The error curves in Figure 1 depict somewhat more conservative error estimates than the values given by Equation (2). This is due to upward rounding of the total position errors on the computer printout from which data shown in Figure 1 were compiled.

Listings of a FORTRAN implementation of Bowring's direct and inverse algorithms are given in Figures 3 and 4, and those of Vincenty's direct and inverse algorithms in Figures 5 and 6.

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Vincenty, T. 1982, private communication

LENGTH OF RADIAL GEODESIC LINE
UP TO WHICH THE ERROR OF BOWRING'S ALGORITHMS
DOES NOT EXCEED THE INDICATED UPPER BOUND

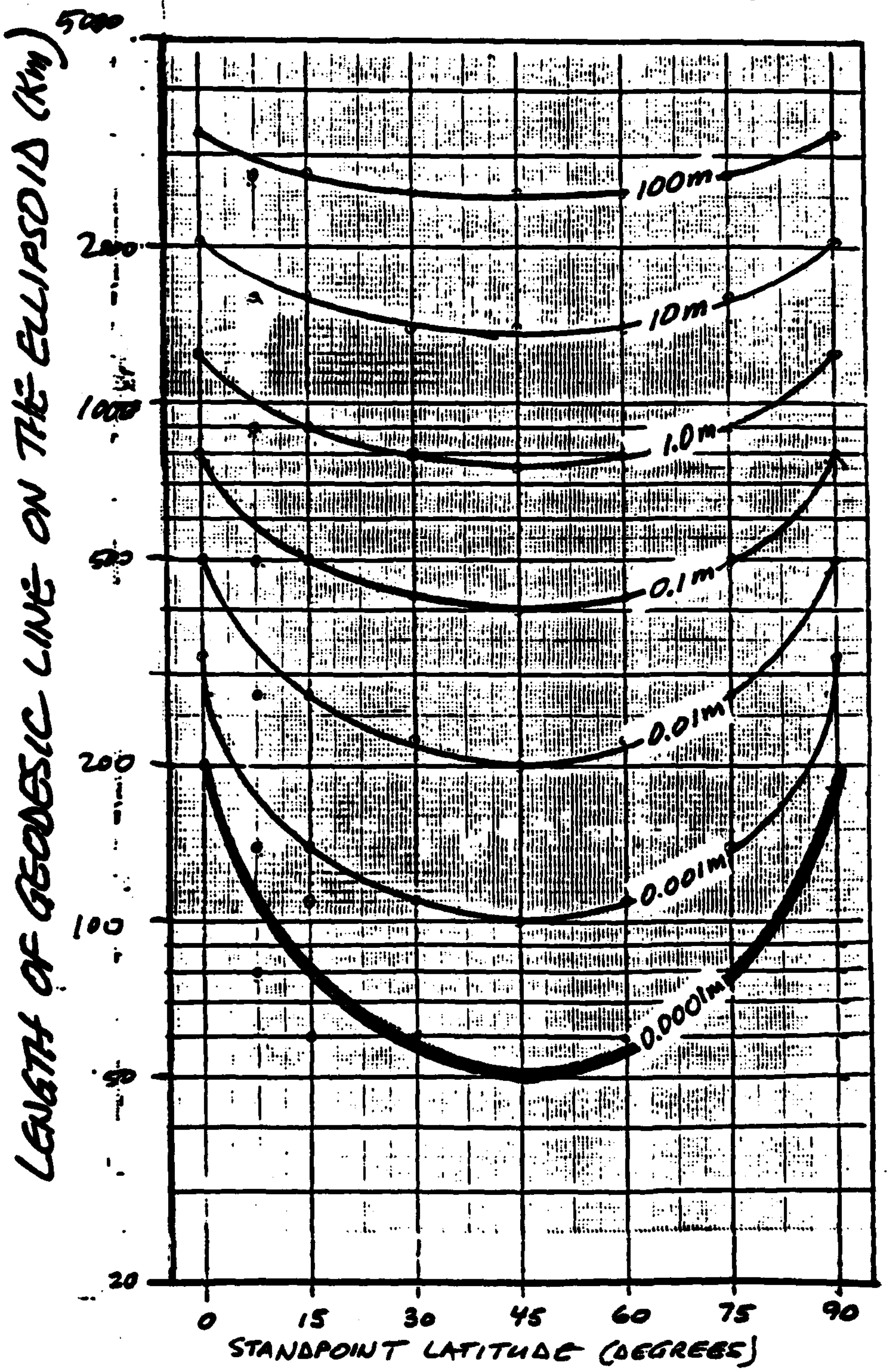

FIGURE 1.

UPPER BOUND OF POSITION ERROR
OF BOWRING'S DIRECT AND INVERSE ALGORITHM:

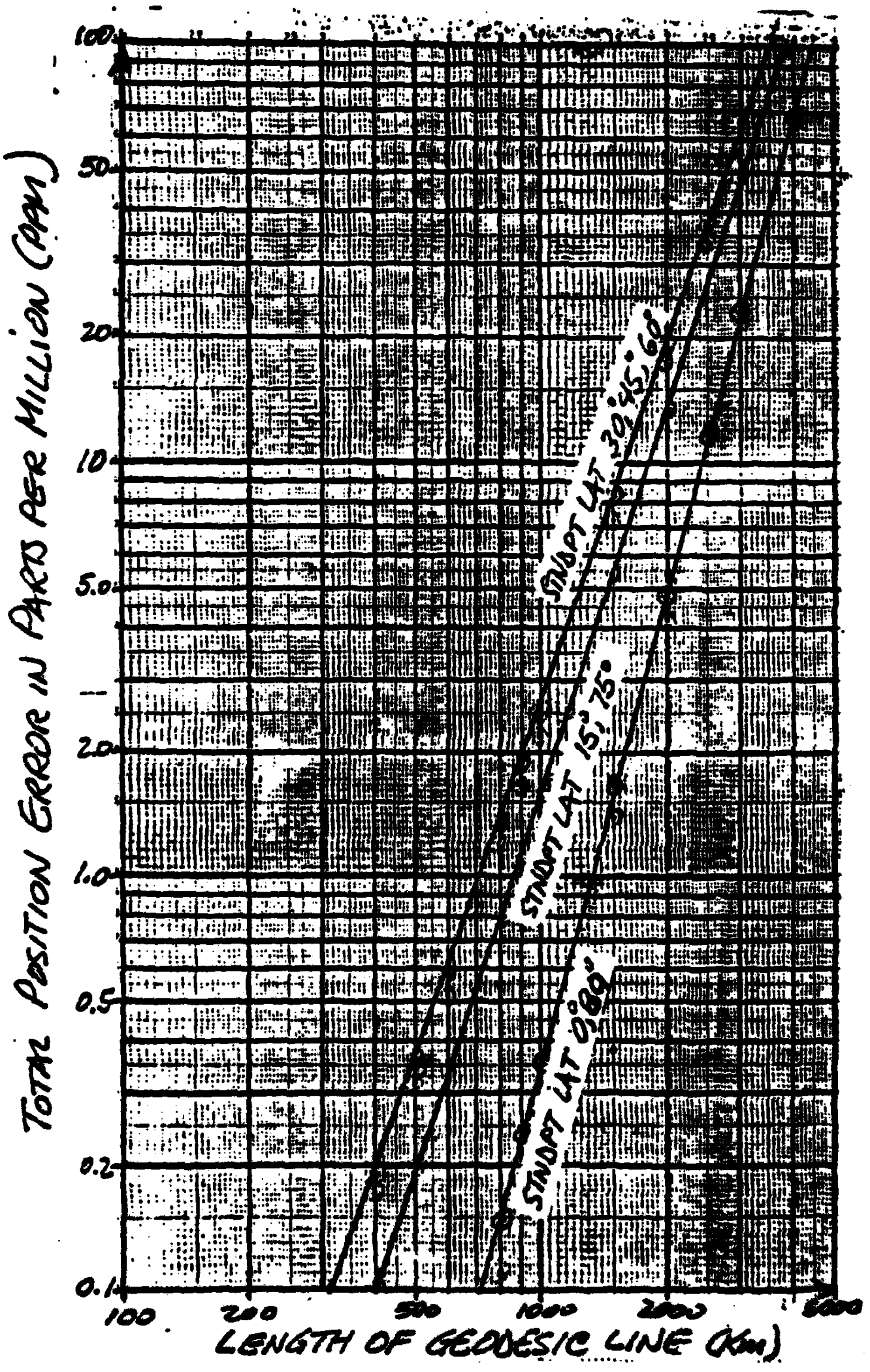

FIGURE 2.

```
      SUBROUTINE GSLDIR(A,FINV,GLAT1,GLON1,FAZ,S,GLAT2,GLON2,BAZ)
C
C *** GEODESIC SHORT-LINE (NONITERATIVE) DIRECT SOLUTION AFTER W. H. BOWRING
C
C *** A IS THE SEMI-MAJOR AXIS OF THE REFERENCE ELLIPSOID
C *** FINV=1/F IS THE INVERSE FLATTENING OF THE REFERENCE ELLIPSOID
C *** LATITUDES AND LONGITUDES IN RADIANS POSITIVE NORTH AND WEST
C *** FORWARD AND BACK AZIMUTHS IN RADIANS CLOCKWISE FROM NORTH
C *** GEODESIC DISTANCE S ASSUMED IN SAME UNITS AS SEMI-MAJOR AXIS A
C
C *** STANDPOINT/FOREPOINT MUST NOT BE THE GEOGRAPHIC POLE
C
C ***  STANDPT  RADIAL LINE LENGTH (KM) UP TO WHICH TOTAL ERROR DOES NOT EXCEED
C *** LATITUDE ----- MM ----   ------ MTRS ------   ----------- PPM -----------
C ***  (DEG)   0.1    1   10   0.1    1   10  100   0.1  0.2    1    2   10   20
C ***   ---    -------------   -------------------   ------------------------------
C ***     0    200  325  500   800 1250 2050 3100   850  950 1400 1650 2450 2950
C ***    15     80  130  275   500  900 1600 2750   450  500  850 1100 1800 2250
C ***    30     60  110  225   425  800 1400 2550   150  425  700  900 1600 2050
C ***    45     50  100  200   400  750 1400 2550   125  400  700  900 1600 2050
C ***    60     60  120  225   450  800 1500 2700   150  450  750 1000 1750 2200
C ***    75     80  150  300   550 1000 1700 2950   500  600 1000 1200 1950 2400
C ***    89    190  300  675   800 1250 2050 3100   800  900 1350 1650 2500 2950
C ***   ---    -------------   -------------------   ------------------------------
C
C *** REFERENCE: SURVEYING AND MAPPING, VOL. 41, NO. 2, 1981, PP. 135-141
C
C *** PROGRAMMED FOR VAX-11/780 BY L. PFEIFER, DMA/GSS FE/AFB WY 82005, 31JAN83
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (PI=3.1415926535897932D0,TWOPI=PI+PI)
C *** OMIT FOLLOWING LINE IF COMPUTATION ALONG MERIDIAN OVER POLE DOES NOT ARISE
      PARAMETER (HALFPI=PI/2.)
      EPS=-1./FINV+1.
      EPS=1./EPS/EPS-1.
      [illegible]
      [illegible]
      SL=DSIN(GLAT1)
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
C *** OMIT FOLLOWING LINE IF COMPUTATION ALONG MERIDIAN OVER POLE DOES NOT ARISE
      IF([illegible].LT.0.3D-11)C=1.
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]*SL*CA
      [illegible](CS[illegible],BAZ)
      [illegible]
      IF(DABS(GLON2).GT.PI)GLON2=GLON2-DSIGN(TWOPI,GLON2)
      [illegible].)*SA*SL/E+CA
      [illegible]
      [illegible]((D*A/3.+GLAT1)*2.)
      [illegible]
      [illegible]
C *** OMIT FOLLOWING LINE IF COMPUTATION ALONG MERIDIAN OVER POLE DOES NOT ARISE
      IF(DABS(GLAT2).GT.HALFPI)GLAT2=DSIGN(PI,GLAT2)-GLAT2
      [illegible]
      [illegible]
      IF(BAZ.LT.0.)BAZ=BAZ+TWOPI
      RETURN
      END
```

Figure 3.

```
      SUBROUTINE GSLINV(A,FINV,GLAT1,GLON1,GLAT2,GLON2,FAZ,BAZ,S)
C
C *** GEODESIC SHORT-LINE (NONITERATIVE) INVERSE SOLUTION AFTER W. H. BOWRING
C
C *** A IS THE SEMI-MAJOR AXIS OF THE REFERENCE ELLIPSOID
C *** FINV=1/F IS THE INVERSE FLATTENING OF THE REFERENCE ELLIPSOID
C *** LATITUDES AND LONGITUDES IN RADIANS POSITIVE NORTH AND WEST
C *** FORWARD AND BACK AZIMUTHS IN RADIANS CLOCKWISE FROM NORTH
C *** GEODESIC DISTANCE S RETURNED IN SAME UNITS AS SEMI-MAJOR AXIS A
C
C *** STANDPOINT/FOREPOINT MUST NOT BE THE GEOGRAPHIC POLE
C
C ***  STANDPT  RADIAL LINE LENGTH (KM) UP TO WHICH TOTAL ERROR DOES NOT EXCEED
C *** LATITUDE ----- MM ----   ------ MTRS ------   ----------- PPM -----------
C ***  (DEG)   0.1    1   10   0.1    1   10  100   0.1  0.2    1    2   10   20
C ***   ---    -------------   -------------------   ------------------------------
C ***     0    225  325  500   800 1100 2100 3350   850  950 1400 1700 2500 3000
C ***    15     80  140  275   550 1000 1700 2850   475  550  950 1200 1950 2400
C ***    30     60  110  225   425  800 1500 2650   350  425  750  950 1750 2150
C ***    45     50  100  200   400  750 1400 2550   300  375  675  900 1550 2000
C ***    60     60  110  225   425  775 1400 2550   325  400  700  900 1600 2000
C ***    75     80  140  275   500  900 1600 2750   425  500  900 1100 1850 2250
C ***    89    180  275  475   775 1300 2000 3250   750  900 1300 1600 2450 2900
C ***   ---    -------------   -------------------   ------------------------------
C
C *** REFERENCE: SURVEYING AND MAPPING, VOL. 41, NO. 2, 1981, PP. 135-141
C
C *** PROGRAMMED FOR VAX-11/780 BY L. PFEIFER, DMA/GSS FE/AFB WY 82005, 31JAN83
C
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER (PI=3.1415926535897932D0,TWOPI=PI+PI)
      EPS=-1./FINV+1.
      EPS=1./EPS/EPS-1.
      SL=DSIN(GLAT1)
      D=-SL*SL+1.
      CL=DSQRT(D)
      S=D*EPS+1.
      B=DSQRT(S)
      C=DSQRT(D*D*EPS+1.)
      D=GLON1-GLON2
C *** OMIT FOLLOWING LINE IF COMPUTATION ALONG MERIDIAN OVER POLE DOES NOT ARISE
      IF(DABS(DABS(D)-PI).LT.0.3D-11)C=1.
      W=-D/2.
      SC=DSIN(D)/C
      CC=DCOS(D)
      D=GLAT2-GLAT1
      D=(DSIN((D/3.+GLAT1)*2.)*D*EPS*3./S+4.)*D/B/8.
      CD=DCOS(D)
      SD=DSIN(D)
      FAZ=SD*CC
      BAZ=(B*CL*CD-SL*SD)*SC
      D=DSQRT(FAZ*FAZ+BAZ*BAZ)
      D=DASIN(D)*2.
      S=DSQRT(EPS+1.)*D*A/S
      D=DATAN((B*CL*SD/CD+SL)*SC/CC)
      BAZ=DATAN2(BAZ,FAZ)
      FAZ=BAZ-D
      IF(FAZ.LT.0.)FAZ=FAZ+TWOPI
      BAZ=BAZ+D-DSIGN(PI,BAZ)
      IF(BAZ.LT.0.)BAZ=BAZ+TWOPI
      RETURN
      END
```

Figure 4.

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